

Exercise 8

Find dy/dx by implicit differentiation.

$$x^3 - xy^2 + y^3 = 1$$

Solution

Differentiate both sides with respect to x .

$$\begin{aligned}\frac{d}{dx}(1) &= \frac{d}{dx}(x^3 - xy^2 + y^3) \\ 0 &= \frac{d}{dx}(x^3) - \frac{d}{dx}(xy^2) + \frac{d}{dx}(y^3) \\ &= (3x^2) - \left[\frac{d}{dx}(x) \right] y^2 - x \left[\frac{d}{dx}(y^2) \right] + \left[3y^2 \cdot \frac{d}{dx}(y) \right] \\ &= 3x^2 - (1)y^2 - x \left[2y \cdot \frac{d}{dx}(y) \right] + 3y^2 y' \\ &= 3x^2 - y^2 - 2xyy' + 3y^2 y'\end{aligned}$$

Solve for y' .

$$y' = \frac{3x^2 - y^2}{2xy - 3y^2}$$